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II. METEOROLOGICAL ASPECTS OF AIR POLLUTION

The influence of the turning of wind with height on crosswind diffusion

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Theoretical indications of the magnitude of horizontal spread resulting from the interaction between vertical diffusion and the wind profile are briefly reviewed. Assuming a fairly realistic form for the spectrum of turbulence the effect of this interaction is evaluated from a treatment by F. B. Smith, and this is combined with the theoretical growth arising directly from the horizontal component of turbulence to give a total crosswind growth curve.

Two series of field experiments on dispersion in stable conditions are examined, one in Sweden using an elevated source of smoke puffs, the other in the U.S.A. using a continuous ground release of a fluorescent tracer. Both indicate that distortion of the plume by the turning of the wind became significant beyond 2–3 km downwind, but that the immediate transference of the effect to enhance the spread at a given level was not important within about 5 km for the elevated source or about 12 km for the ground source.

1. Introduction

The main purpose of this note is the consideration of the direct observational evidence for the effect of the change of wind direction with height in producing crosswind dispersion of a plume of windborne material, extra to that which arises directly from the crosswind component of turbulence. There is no lack of visual evidence of the systematic shearing of plumes or clouds of material in the lower atmosphere, and theoretically there seems no doubt about the *ultimate* dominance of the wind shear contribution to spread at a given level, provided the vertical growth is maintained. However, the time of travel or distance downwind beyond which this dominance applies has not been precisely defined. After a reconsideration of theoretical indications two independent series of diffusion experiments are re-examined for evidence on this point.

2. Theoretical aspects

The first analysis of the effect of shear on horizontal dispersion in the atmosphere was provided by Saffman (1962), using the simple diffusion model in which the transfer of material across any plane is specified by the product of the gradient of concentration (normal to the plane) and an eddy diffusivity K. The solutions then obtained were actually for the alongwind spread (specified as a variance of particle spread σ_x^2) arising solely from the variation of mean *speed* $\bar{u}$ with height. Saffman first considered a vertically bounded flow of depth h (analogous to the flow in a pipe, first analysed in the present context by Taylor (1953, 1954)) and gave an asymptotic solution for large time of travel T. With K_z = constant and a linear wind profile $\bar{u} = 2Uz/h$ the result for dispersion at ground level was

$$\sigma_x^2 = U^2h^2T/15K_z. \tag{1}$$

For a source on the ground, but with no upper bound to the vertical spread, and assuming $\bar{u} = \alpha z$, the corresponding result was

$$\sigma_x^2 = 0.036\alpha^2K_zT^3. \tag{2}$$

A third solution, for vertical spread which is unbounded both upward and downward, was subsequently provided by Smith (1965) again for the case of $\bar{u} = \alpha z$, as

$$\sigma_x^2 = \tfrac{1}{6}\alpha^2 K_z T^3. \tag{3}$$

These solutions bring out two important points. First, the dependence on the rate of vertical transport (K_z) is in opposite senses according as the vertical spread is limited in both directions or unlimited in at least one direction. Secondly, in the latter condition there is a much more rapid growth of σ_x^2, with T^3 instead of T in the former condition.

The foregoing solutions, valid only for constant K_z and large T, do not provide any indication of the early development of the wind shear effect and therefore cannot immediately be used to specify the distance at which the effect becomes dominant over that of crosswind turbulence *per se*. A possible extension of the simple diffusion model in this direction is the use of numerical solutions of the equation of diffusion as advocated by Tyldesley & Wallington (1965). A different approach, which is more revealing in certain respects, was adopted by Smith (1965), and will be followed here. This approach uses a purely kinematical (statistical) representation of particle motion, in terms of autocorrelation and spectral functions. A homogeneous field of turbulence is assumed, with the mean speed $\bar{u}$ (x-direction) constant with height, but the mean direction varying with height to give a linear variation with height of $\bar{v}$ (y-direction). The crosswind dispersion σ_y^2 at a given level of particles released sequentially from a fixed point is expressed in three terms, valid for any time of travel T, as follows:

$$\left.\begin{aligned} \sigma_y^2 = \sigma\;{}^2 &\int_0^\infty G_{\mathrm{L}}(n)\left(\frac{\sin r}{r}\right)^2 \mathrm{d}n && (a)\\ +\tfrac{1}{4}\sigma_{\mathrm{w}}^2\psi^2T^4 &\int_0^\infty F_{\mathrm{L}}(n)\left(\frac{\sin r - r\cos r}{r^2}\right)^2 \mathrm{d}n && (b)\\ -\rho^2\sigma_{\mathrm{v}}^2T^2 &\int_0^\infty F_{\mathrm{L}}(n)\left(\frac{\sin r}{r}\right)^2 \mathrm{d}n, && (c) \end{aligned}\right\} \tag{4}$$

where $r = \pi nT$, ψ is $\mathrm{d}\bar{v}/\mathrm{d}z$, σ_{w}, σ_{v} are the root mean square fluctuations of the vertical and crosswind components w' and v', $F_{\mathrm{L}}(n)$, $G_{\mathrm{L}}(n)$ are the corresponding normalized Lagrangian spectrum functions, and ρ is the correlation coefficient for the vertical and crosswind velocities, i.e. $\overline{w'v'}/\sigma_{\mathrm{w}}\sigma_{\mathrm{v}}$.

Term (a) is the direct effect of the horizontal component of turbulence (no effect of wind shear involved), which for large T tends to $2\sigma_{\mathrm{v}}^2 t_{\mathrm{vL}} T$ where t_{vL} is the integral time-scale for the crosswind component.

Term (b), in which no effect of horizontal turbulence is involved, is the effect of the systematic wind shear. Essentially it represents the effect of the particle taking up instantaneously the mean horizontal velocity at each level. The weighting function applied to $F_{\mathrm{L}}(n)$ is a narrow filter with a maximum value (0.19) at $\pi nT = 2$. For large T this filter transmits only very low frequencies, for which $F_{\mathrm{L}}(n)$ is effectively constant and equal to $4t_{\mathrm{wL}}$ where t_{wL} is the integral time-scale for the vertical component. Accordingly, for large T, this term tends to $\frac{1}{6}\psi^2\sigma_{\mathrm{w}}^2 t_{\mathrm{wL}} T^3$ or $\frac{1}{6}\psi^2 K_z T^3$ (on writing $K_z = \sigma_{\mathrm{w}}^2 t_{\mathrm{wL}}$), which is identical with (3) on substituting $\psi = \alpha$.

Term (c) represents the effect of that part of the horizontal component which is correlated with the vertical component. It acts in opposition to term (b), being a reflexion of the extent to which the particle fails to adjust its horizontal velocity instantaneously to the mean velocity at its level. At large T its magnitude is $-2\rho^2\sigma_{\mathrm{v}}^2 t_{\mathrm{wL}} T$ (with ρ normally < 1).

It is obvious that at sufficiently large T term (*b*) must become dominant. At small enough T, on the other hand, this term becomes very small and the resultant of terms (*a*) and (*c*) becomes dominant (except in the special case of $\rho = 1$). It is especially interesting to note that the direct effect of the crosswind component is apparently partially neutralized by the effect of correlation between w' and v'. The real point is that some of the crosswind displacement due to the crosswind component occurs in the form of an initial shearing of the distribution of particles, upward moving particles tending to be displaced to one side and downward moving particles to the other, but in the sense opposite to the shearing which ultimately sets in and which is represented by term (*b*).

3. Application of Smith's solution for general values of T

Smith's solutions may be carried further once the functions $F_L(n)$ and $G_L(n)$ have been specified. Here these functions are assumed to have a common form

$$nF_L(n) = nG_L(n) = \frac{N}{(1+\frac{3}{2}N)^{\frac{5}{3}}}, \tag{5}$$

where

$$N = n/n_m = 4t_L n, \tag{6}$$

though with n_m (the frequency for maximum $nF(n)$ or $nG(n)$) and t_L having different values for the vertical ($F(n)$) and crosswind ($G(n)$) components. Although this form has been demonstrated to give a good fit to *fixed point* frequency spectra of the vertical component (Busch & Panofsky 1968), its adoption in the present context is questionable on two grounds. In the first place it has not

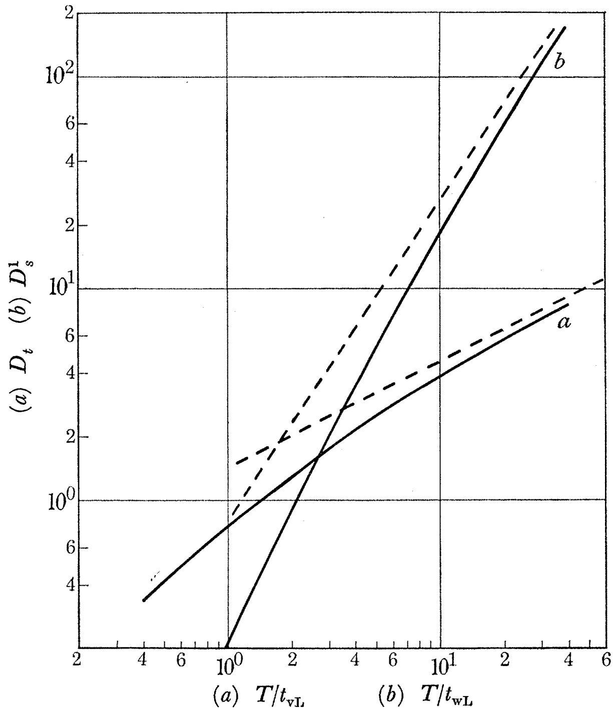

Figure 1. Theoretical crosswind spreads in unbounded shear flow.

$$\left.\begin{aligned} D_t &= \sigma_{yt}/\sigma_v t_{vL},\ D_s = \sigma_{ys}/\sigma_v t_{vL} \\ D_s^1 &= D_s/\tfrac{1}{2}\psi t_{vL}\frac{\sigma_w}{\sigma_v}\frac{t_{wL}^2}{t_{vL}^2} \end{aligned}\right\} \text{broken lines are asymptotic limits}$$

been verified as a good fit to v-component spectra. In the second place the required spectral functions are *Lagrangian* in character, whereas the fixed point spectra are regarded as essentially *Eulerian*. However, the precise shape of the spectral function is probably not as important as the actual magnitude of the integral time scale, and on this basis it is considered that an enlightening examination of terms (a) and (b) may be made by means of (5) as a tolerable approximation.

Graphical integrations gave results which are plotted in non-dimensional form in figure 1, using the following relations for convenience, in which the subscripts t and s are used to designate the direct effect of the horizontal component of turbulence and the indirect effect of the wind shear.

$$D_t = \sigma_{yt}/\sigma_v t_{vL}, \tag{7}$$

$$D_s = \sigma_{ys}/\sigma_v t_{vL}, \tag{8}$$

$$D'_s = D_s \bigg/ \left(\tfrac{1}{2}\psi t_{vL} \frac{\sigma_w}{\sigma_v} \frac{t^2_{wL}}{t^2_{vL}}\right). \tag{9}$$

From the asymptotic (large T) limits of D_t and D'_s which are also shown an idea may immediately be obtained of the error which would arise from assumption of these limiting forms at relatively small values of T/t_L.

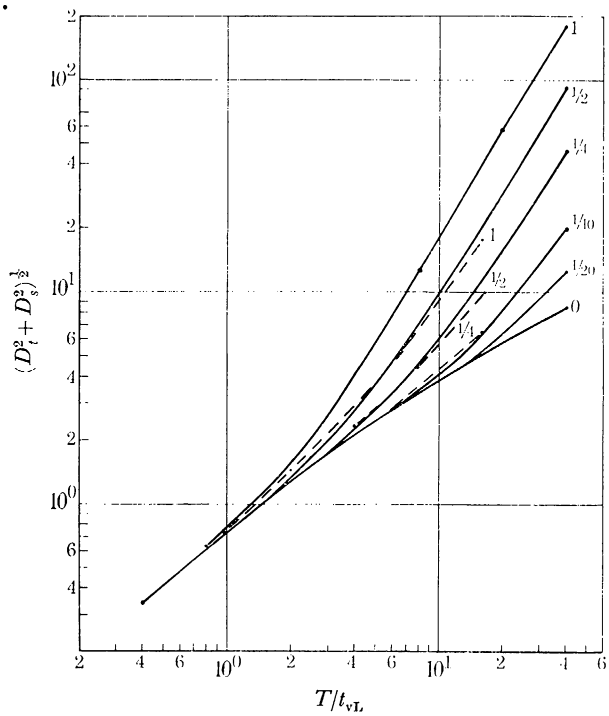

FIGURE 2. Resultant spread in unbounded shear flow. $D = \sigma_y/\sigma_v t_{vL}$. Numbers are values of $\frac{1}{2}\psi t_{vL}\sigma_w/\sigma_v$. ——, $t_{vL}/t_{wL} = 1$; – – –, $= 8$.

Compounding of the effects of turbulence and mean shear requires the specification of ψt_{vL}, σ_w/σ_v and t_{wL}/t_{vL}. The net effect, given by $D = (D_t^2 + D_s^2)^{\frac{1}{2}}$, is displayed in figure 2 for a convenient range of values of $\frac{1}{2}\psi t_{vL}(\sigma_w/\sigma_v)$ and for t_{vL}/t_{wL} equal to 1 or 8. Note that the curves show régimes in which turbulence and shear are respectively dominant, with a fairly sharp transition between the two. Note also that the effect of correlation between w' and v' is excluded. As this (term (c) in (4)) is otherwise of the same form as term (a) the error can be regarded as effectively an over-estimation of the D_t contribution and hence an overestimation of the time taken for the D_s term to become dominant.

Evidently an important effect of wind shear should be apparent in a fairly abrupt change in the σ_y, T relation from a region of slightly less than linear growth to one of distinctly more than linear growth. The interesting point is that hitherto the analysis of dispersion data over extensive ranges of distance has usually indicated a slightly less than linear growth. The only notable exceptions have been for the case of a relatively small cluster of particles, when the acceleration of spread due to progressive influence of eddies of larger size leads to a faster than linear growth. So far no diffusion experiments have been reported as designed specifically to examine the relative effect of turbulence and shear, but two independent experimental studies have recently been found to contain information of interest in this connexion.

4. Högström's data on diffusion from an elevated source

Högström (1964) has published data on the behaviour of elongated puffs of smoke generated over periods of 30 s at an elevated position on a chimney or tower in Sweden. Photographs taken from near the point of release were used to measure two aspects of the crosswind and vertical dispersion—the scattering of the centres of successive puffs about their mean position (expressed as a standard deviation, σ_{y_1}, in Högström's notation)—and the visible sizes of the individual puffs. From the visible dimensions as a function of distance it is possible to deduce the equivalent standard deviation σ_{y_r} of particles relative to the centre of the puff provided three conditions hold:

(*a*) The visible outline of the puff photograph represents a contour on which the line-of-sight integrated concentration is a constant for given optical conditions.

(*b*) The distribution of particles about the puff centre is Gaussian.

(*c*) The observations show the puff size passing through a maximum at some distance.

In Högström's measurements the observational requirement was satisfied and the first two assumptions, though questionable, were as usual adopted as reasonable approximations.

Data for σ_{y_1}, for releases at a height of 87 m at Studsvik, are summarized in table 1 and figure 6 of the original paper. Apparently there was no obvious effect of stability as represented by the parameter

$$\lambda = \lg\left[10^5\left(\frac{\partial T_p}{\partial z}\right)\Big/ u_f^2\right], \tag{10}$$

where $\partial T_p/\partial z$ is the average vertical gradient of potential temperature derived from measurements at heights of 30 and 122 m, and u_f is the wind at a height of 500 m interpolated from regular upper air soundings. The average data for σ_{y_1} show onset of a more rapid than linear growth at a distance between 2.5 and 3 km. Note, however, that σ_{y_1} is derived from crosswind displacements of puffs irrespective of their level relative to the source. Thus it includes a contribution from the *systematic* displacement of puffs to opposite sides above and below the release level—corresponding to the bodily distortion of a continuous plume—whereas this contribution is not included in (*b*) of (4). On the other hand, σ_{y_1} excludes the contribution from the growth of puffs (σ_{y_r})—which would of course be included in a continuous plume. To make proper comparison of Högström's data with theoretical expectation it is necessary to subtract the contribution associated with 'plume distortion' and to add the σ_{y_r} contribution.

The values of σ_{y_r} were not included in the original paper in the same format as those of σ_{y_1} but have since been obtained from the author. They show two interesting differences from σ_{y_1}—they are fairly sensitive to stability—and they do not show on average any obvious sign of the development of a growth faster than linear, up to the maximum distance of 5 km. For present purposes use has been made of two groups of σ_{y_r} data corresponding to neutral conditions and to the most stable conditions, represented by $\lambda \geqslant 2.0$. These have been combined with σ_{y_r} to give the resultant spread $(\sigma^2_{y_1}+\sigma^2_{y_r})^{\frac{1}{2}}$ (see table 1) and the latter is shown graphically in figure 3.

TABLE 1. HÖGSTRÖM'S DATA ON CROSSWIND SPREAD (STUDSVIK, 87 m LEVEL)

distance/km		0.5	1	1.5	2	2.5	3	4	5
σ_{y_1}/m		32	60	88	113	134	192	290	399
σ_{y_r}	neutral	24	46	63	77	91	103	125	145
	stable ($\lambda \geqslant 2.0$)	11	19	27	33	38	43	51	68
$(\sigma^2_{y_1}+\sigma^2_{y_r})^{\frac{1}{2}}$	neutral	40	76	108	137	162	218	316	425
	stable	33	63	92	117	139	197	295	403

Notes. The σ_{y_1} values are averages of data given in table 1 of Högström's (1964). The σ_{y_r} values were supplied in a private communication.

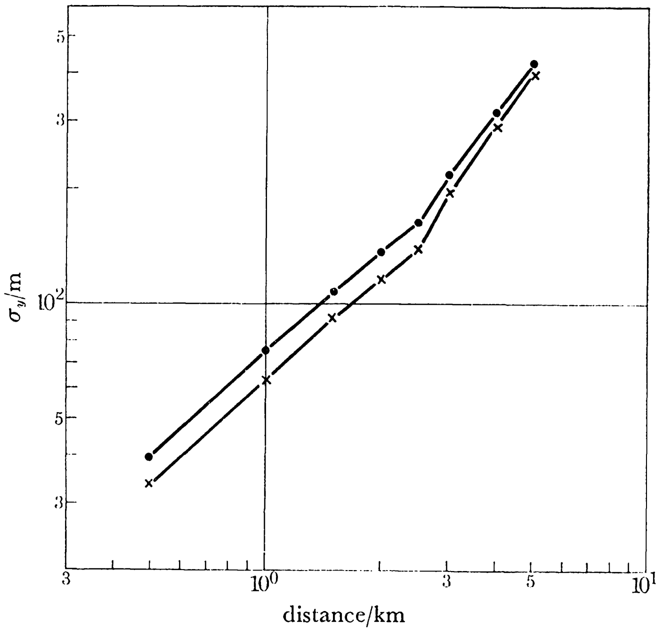

FIGURE 3. Experimental data on crosswind spread from an elevated source (Högström 1964). $\sigma^2_y = \sigma^2_{y_1}+\sigma^2_{y_r}$ in Högström's notation. ●, neutral; ×, stable ($\lambda \geqslant 2.0$).

Figure 3 shows a well defined change in the growth curve at a distance of about 2.5 km. However, as already noted, this growth curve contains the effect of 'plume distortion', and since this must precede the contribution to growth at *a given level* the only immediate conclusion which can be drawn is that this latter contribution must set in effectively at some greater distance. To proceed any further it is necessary to estimate the separate contributions at say the maximum distance of 5 km. The first step is to estimate the contribution from the crosswind component of turbulence alone at 5 km by extrapolating the first sections of the growth curves in figure 3. This can be achieved in an objective manner by a trial and error fitting with curve (*a*) of figure 1 to derive the correspondence between distance x and T/t_{vL}. Subtraction (of squares) of the extrapolated turbulence contribution from the observed total yields the total contribution from shear. To estimate the part which is effective at a given level use may be made of the result (following Smith) for large T, that this is exactly one-half of the total, noting that the procedure may be expected to lead to an *overestimate* at practical values of T. The calculations are summarized in table 2. From this it is seen that at 5 km the *overestimated* increase in spread (at a given level) as a result of shear is only 14 % in neutral conditions and 22 % in stable conditions.

It is particularly interesting that Högström concludes σ_{y_1} to be independent of stability, and the slight difference in the two estimates of the relative effect of shear arises from the pronounced effect of stability on σ_{y_r}. Although the absence of stability influence on σ_{y_1} is demonstrated (in Högström's table 1) for most of the range of distance it is not definitely demonstrated at 4 and 5 km, since at these distances the data are confined to a narrow range of λ. In terms of (4) (term (*b*)) an absence of stability influence would require compensating stability effects on ψ and σ_w. This seems reasonable qualitatively, and according to Högström there was some quantitative support from his values of ψ and measured vertical spread. If this absence of net stability effect could be more generally demonstrated it would of course greatly simplify the specification of the effect of wind shear on crosswind spread.

Table 2. Evaluation of effect of shear on crosswind spread at 5 km

(Högström's data, Studsvik, 87 m level)

	neutral	stable
ratio of $\sigma = (\sigma_{y1}^2+\sigma_{yr}^2)^{\frac{1}{2}}$ values at 2.5 and 1 km (from table 1)	2.14	2.21
values of T/t_L to give same ratio (from curve (*a*) of figure 1)	1, 0.4	0.8, 0.32
extrapolated value of σ at 5 km (turbulence only), A	285	236
actual value of σ at 5 km (from table 1), B	425	403
total contribution from shear (including overall distortion), $C = (B^2-A^2)^{\frac{1}{2}}$	315	326
estimate of contribution at one level from shear, $D = \frac{1}{2}C$	157	163
total at one level (turbulence + shear), $(A^2+D^2)^{\frac{1}{2}}$	326	287
percentage increase as a result of shear	14	22

5. The Hanford 'greenglow' data on diffusion from a point source at ground level in stable conditions

The second collection of data to be considered here comes from measurements made at night time on the Hanford (Washington) reservation of the United States Atomic Energy Commission. By the use of a fluorescent pigment as a tracer, crosswind distributions of concentration were measured near the ground at various distances up to 25.6 km. Some crosswind distributions were also measured with samplers mounted on towers but then only at distances up to 3.2 km. The tower results from one of the earlier experiments (no. 3, 28 June 1959), discussed by Barad & Fuguay (1962), bring out very clearly the first stage of the effect of wind shear. The cross-section of the plume in a vertical plane normal to the general direction of travel was found to develop a systematic distortion in the sense expected from the prevailing turning of wind with height. Over distances up to 0.8 km at least the crosswind spread evidently grew at a slower than linear rate. Between 1.6 and 3.2 km the σ_y of the *whole* cross-section (i.e. including the distortion) increased by a factor of 2.8 (see table 2 of Barad & Fuguay 1962), but there is no sign of a faster than linear growth in the σ_y at a given level. Here evidently was the effect of wind shear setting in but not yet significantly communicated to the spread at a given level.

An overall summary of the data on crosswind distribution at ground level has been given by Fuguay, Simpson & Hinds (1964). There are 16 experiments for which σ_y is tabulated at distances up to 25.6 km. The increase in σ_y over the fourfold distances 3.2 and 12.8 km is more than fourfold in only four of sixteen cases. On the other hand, the increase over the next (twofold) interval 12.8 and 25.6 km is more than twofold in nine of eleven cases. Thus the effect of shear on spread at a given level appears to have set in significantly at about 12.8 km. The average result is shown in figure 4. Here the values of σ_y were first normalized by dividing by σ_θ (the standard deviation of the fluctuation of wind direction θ) at a height of 2.1 m during the periods of release. Also plotted in figure 4 is a curve derived from (*a*) of figure 1, using the definition of D_t in (7), as follows

$$\frac{\sigma_y}{\sigma_\theta} \simeq \frac{\bar{u}\sigma_y}{\sigma_v} \simeq \bar{u}t_{vL}\,D_t. \tag{11}$$

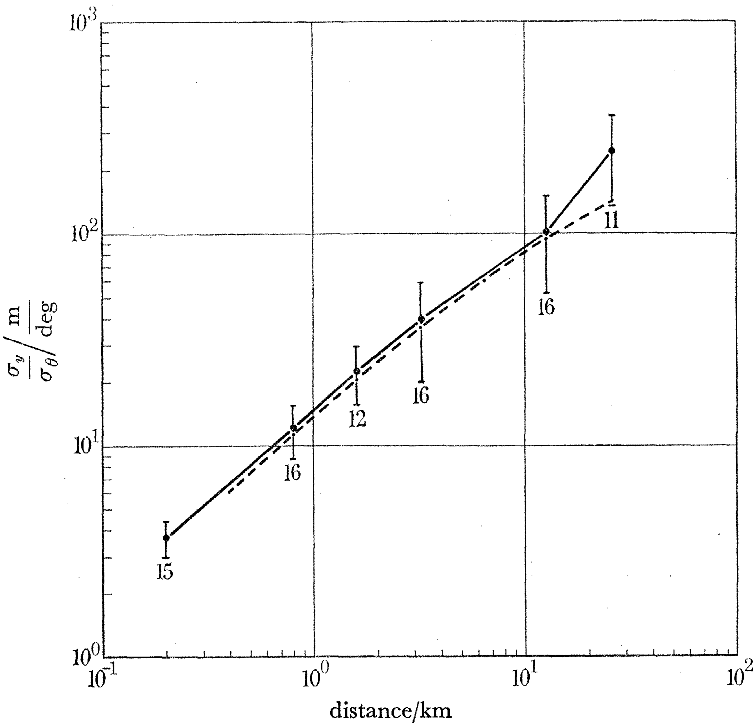

FIGURE 4. Hanford experimental data on crosswind spread from a ground source in stable conditions (solid line). Vertical lines are r.m.s. deviations for the number of values indicated. Broken line is the theoretical value for flow without shear.

The shapes of curve (*a*) of figure 1 and of the observed curve in figure 4 match if distance $x = 1600$ m and $T/t_{vL} = 1$ are identified. Accordingly, writing $x = \bar{u}T$, we have

$$\bar{u}t_{vL} \simeq x \simeq 1600. \tag{12}$$

The curve of σ_y/σ_θ obtained in this way (plotted with σ_θ in degrees) is seen to fit very closely as regards absolute magnitude, up to the penultimate distance 12.8 km.

The rather remarkable correspondence of the observed and calculated values of σ_y/σ_θ up to 12.8 km is probably fortuitous to some extent, especially in that the representation of σ_θ is deficient in two respects. First, the values quoted are for an averaging time of 20 s and on that account will be too low because of suppression of relatively high-frequency contributions, though this error is unlikely to be more than a small percentage in view of the implied time scale (t_{vL}) of some hundreds of seconds. A possibly more serious uncertainty is the variation of σ_θ with height. There is also an uncertainty from the implicit assumption in (11) and (12) that the effective speed of travel of the tracer is the same as the mean wind speed at a height of 2.1 m. However, these qualifications about the absolute fit at distances up to 12.8 km do not detract from the indication that on average the effect of shear became important only after this distance. As in the analysis of Högström's data the first part of the curve may be extrapolated, using the form of curve (*a*) of figure 1, to give the contribution from turbulence alone at 25.6 km and, by variance difference, the separate contribution from the shear. From this it appears that the resultant spread (σ_y/σ_θ) of 246 m/degree is composed of respective contributions 140 and 203 m/degree, i.e. here the shear contribution is dominant and for practical purposes the direct contribution from turbulence could be disregarded.

6. Concluding remarks

The field measurements of crosswind dispersion in stable conditions considered here indicate quite clearly that a bodily crosswind distortion of the plume from a point source (either elevated or on the ground) sets in between 2 and 3 km. However, the *form* of the crosswind growth curves suggests strongly that the communication of the distortion to the spread at a given level was not of practical importance at distances below about 5 km in the case of the elevated source and about 12 km in the case of the ground level source. Thereafter the implication is that the shear contribution is dominant. Additional support is provided by the fact that the *absolute* spread up to 12 km from the ground level source is in close agreement with the predicted effect of the crosswind component of turbulence alone. Complete verification would of course require the additional demonstration that the *absolute contribution* from shear can be predicted. A really critical test of this does not so far appear possible, partly because of inadequate supporting data and partly because the relation given here for general time of travel applies only in homogeneous turbulence. While this condition might be assumed to be a reasonable approximation for the horizontal component of turbulence it is most unlikely to be so for the vertical component.

The numerical and graphical work was done by Mrs A. P. Adeline. I am indebted to Dr U. Högström for the unpublished data included in table 1. The note is published by permission of the Director-General of the Meteorological Office.

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